ACE Network Subject Information Guide

Nonlocal diffusion, theory and applications

Semester 1, 2020

Administration and contact details

Host Department	Department of Mathematics and Statistics
Host Institution	University of Western Australia
Name of lecturer	Serena Dipierro
Phone number	+61 8 6488 3315
Email Address	serena.dipierro@uwa.edu.au
Homepage	https://research-repository.uwa.edu.au/en/persons/serena-dipierro
Name of Honours coordinator	Gopalan Nair
Phone number	+61 8 6488 3377
Email Address	gopalan.nair@uwa.edu.au

Subject details

Start date:	**27 February, 2020**
End date:	**21 May, 2020**
Contact hours per week:	**Monday h 9-10am (Western Australia time zone)**
Lecture day and time:	**Thursday h 12-1pm (Western Australia time zone)**

Subject content

1. **Subject content description**

   In this course, we present some important features focused on nonlocal and fractional diffusions, with special emphasis on problems endowed with a geometric motivation and possessing important applications in physics, biology, engineering, and material sciences.

   The course is planned as a combination of classical topics (perimeter functional, minimal surfaces, curvature flows, partial differential equations) and subjects of emerging research (long-range interactions, memory effects, nonlocal minimal surfaces, nonlocal mean curvature flows).

The structure of the whole unit is based on a dynamic interplay between the development of a solid and rigorous mathematical theory and the attention to a number of concrete applications coming from real world phenomena.

Possible references are:

Nicola Abatangelo, Enrico Valdinoci, *Getting acquainted with the fractional Laplacian*. In *Contemporary research in elliptic PDEs and related topics*, 1–105, Springer INdAM Ser., 33, Springer, Cham, 2019.

Claudia Bucur, Enrico Valdinoci, *Nonlocal diffusion and applications*. Lecture Notes of the Unione Matematica Italiana, 20. Springer, [Cham]; Unione Matematica Italiana, Bologna, 2016. xii+155 pp. ISBN: 978-3-319-28738-6; 978-3-319-28739-3

Matteo Cozzi, Alessio Figalli, *Regularity theory for local and nonlocal minimal surfaces: an overview*. In *Nonlocal and nonlinear diffusions and interactions: new methods and directions*, 117–158, Lecture Notes in Math., 2186, Fond. CIME/CIME Found. Subser., Springer, Cham, 2017.

Serena Dipierro, Enrico Valdinoci, *Nonlocal minimal surfaces: interior regularity, quantitative estimates and boundary stickiness*. In *Recent developments in nonlocal theory*, 165–209, De Gruyter, Berlin, 2018.

2. Week-by-week topic overview

The course is scheduled for 1 hour per week, for 12 weeks (in case some students need additional workload to obtain credit from their own university, this flexibility can also be arranged according to their requests).

Nonlocal equations (weeks 1-7)

Week 1: definition of fractional Laplacian
Week 2: similarities and differences between cassical and fractional equations
Week 3: all functions are s-harmonic up to a small error
Week 4: applications to water wave models
Week 5: applications to atom dislocations in crystals
Week 6: applications to phase transition models
Week 7: applications to population dynamics and mathematical biology

Nonlocal minimal surfaces (weeks 8-12)

Week 8: classical and nonlocal perimeter functionals
Week 9: interior regularity

Week 10: boundary behaviour and stickiness
Week 11: geometric flows, applications to image reconstruction
Week 12: Gamma-convergence, connection to phase transition models, and open research problems.

3. Assumed prerequisite knowledge and capabilities

Basic and multivariable calculus.

4. Learning outcomes and objectives

Learning the basics of nonlocal equations, with special focus on the ones driven by the fractional Laplacian. Getting acquainted with nonlocal operators and nonlocal minimal surfaces, developing some intuition about long-range interactions. Understanding the possible applications, combining rigorous mathematics and modelling of concrete situations.

5. Learning resources

Nicola Abatangelo, Enrico Valdinoci, *Getting acquainted with the fractional Laplacian*. In *Contemporary research in elliptic PDEs and related topics*, 1–105, Springer INdAM Ser., 33, Springer, Cham, 2019.

Claudia Bucur, Enrico Valdinoci, *Nonlocal diffusion and applications*. Lecture Notes of the Unione Matematica Italiana, 20. Springer, [Cham]; Unione Matematica Italiana, Bologna, 2016. xii+155 pp. ISBN: 978-3-319-28738-6; 978-3-319-28739-3

Matteo Cozzi, Alessio Figalli, *Regularity theory for local and nonlocal minimal surfaces: an overview*. In *Nonlocal and nonlinear diffusions and interactions: new methods and directions*, 117–158, Lecture Notes in Math., 2186, Fond. CIME/CIME Found. Subser., Springer, Cham, 2017.

Serena Dipierro, Enrico Valdinoci, *Nonlocal minimal surfaces: interior regularity, quantitative estimates and boundary stickiness*. In *Recent developments in nonlocal theory*, 165–209, De Gruyter, Berlin, 2018.

6. Assessment

The assessment is based on weakly assignments that will be turned in by the end of week 10 (in order to have two lectures to discuss the exercises in class). No intermediate tests or final exams are expected for this course.