ACE Network Subject Information Guide

Algebraic Number Theory

Semester 2, 2020

Administration and contact details

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Host Institution	University of Newcastle
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Subject details

Handbook entry URL	TBC
Subject homepage URL	TBC
Honours student hand-out URL	TBC
Start date:	20 July 2020
End date:	23 October 2020
Contact hours per week:	2
Lecture day and time:	TBC in consultation with students
Description of electronic access arrangements for students (for example, WebCT)	A course website/shared directory will be set up containing all course notes etc.

Subject content

1. **Subject content description**

Number theory is the study of integers. However, in order to study questions about the integers, one is often forced to study more general sets numbers. For example, in order to determine which prime numbers can be written as the sum of two squares, p = x^2+y^2, one really needs to consider numbers of the form x+iy, which are irrational but algebraic.

Algebraic Number Theory is thus the study of algebraic numbers (i.e. solutions to polynomial equations with integer coefficients). These are elements in number fields, i.e. finite extensions of the field of rational numbers, for example Q(i), the Gaussian numbers.

Each such number field contains a subring of algebraic integers (e.g. Z[i], the Gaussian integers in our example above) and we're interested in arithmetic in these rings of algebraic integers. In the Gaussian integers, every element can be factorised uniquely into a product of prime elements. However, in many other examples, this unique factorisation fails, for example in the ring Z[sqrt(-5)].

This obstacle can be overcome by moving from elements to ideals – it turns out that every ideal in an algebraic number ring can be factorised uniquely into a product of prime ideals.

This is the starting point for a very rich theory in which we will study these rings (more precisely, a class of rings called Dedekind domains), their groups of units and ideal classes modulo principal ideals, which form a finite group called the ideal class group.

Number theory is famous for borrowing techniques from all other branches of mathematics. The exact topics we will study will determine the techniques we will use, and will be decided based on the interests and backgrounds of the students.

2. Week-by-week topic overview

Topics covered will include the following:

- Number fields
- Dedekind domains
- Minkowski's geometry of numbers and finiteness of the ideal class group
- Dirichlet's Unit Theorem.
- Many examples throughout, especially from quadratic fields and cyclotomic fields.

Optional topics may include, depending on student interest:

- Analytic number theory: zeta functions and the distribution of primes
- L-functions, primes in arithmetic progressions and Dirichlet's class number formulas
- Valuation theory and p-adic numbers
- Galois theory in number fields
- Algebraic function fields
- Elliptic curves

3. Assumed prerequisite knowledge and capabilities

A basic understanding of groups, rings, fields and ideals, such as is taught in a standard undergraduate Abstract Algebra course is assumed.
A first course in Complex Analysis is required for some of the optional topics.
A first course in Number Theory would be helpful, but is not necessary.

4. Learning outcomes and objectives

- Demonstrate an understanding of the content and context of algebraic number theory
- Apply advanced mathematical problem solving skills
- Use sophisticated mathematical communication skills in the presentation of mathematical arguments

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below
Insert Program Learning Outcome here	Choose from list below

Learning Outcome Descriptors at AQF Level 8
Knowledge
K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2: knowledge of research principles and methods
Skills
S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3: cognitive skills to exercise critical thinking and judgement in developing new understanding
S4: technical skills to design and use in a research project
S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences
Application of Knowledge and Skills
A1: with initiative and judgement in professional practice and/or scholarship
A2: to adapt knowledge and skills in diverse contexts
A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters
A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

A number of excellent textbooks cover the material presented in this course, we will rely on the following two:

- Paul Pollack: “A Conversational Introduction to Algebraic Number Theory”, AMS Student Mathematical Library, vol 84.
- Pierre Samuel: “Algebraic Theory of Numbers”, Dover

More advanced textbooks include:

- Serge Lang: “Algebraic Number Theory”, Springer Graduate Texts in Mathematics vol 110.
- Jürgen Neukrich: “Algebraic Number Theory”, Springer
- James Milne: “Algebraic Number Theory”, free course notes at https://www.jmilne.org/math/CourseNotes/ant.html

6. Assessment

Exam/assignment/classwork breakdown					
Exam	50 %	**Assignment**	50 %	**Class work**	0 %
Assignment due dates		21 August 2020	25 September 2020	Click here to enter a date.	Click here to enter a date.
Approximate exam date				October/November 2020	

Institution Honours program details

Weight of subject in total honours assessment at host department	10 of 80
Thesis/subject split at host department	30 of 80
Honours grade ranges at host department:	
HD	85-100 %
D	75-84 %
C	65-74 %
P	50-64 %